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MTP-AERO-63-28 April 23, 1963

GEORGE C. MARSHALL SPACE

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HUNTSVILLE, ALABAMA

A NOTE ON THE SOLUTION FOR THE SLOWLY OSCILLATING BODY OF REVOLUTION IN SUPERSONIC FLOW

Вy

Maximilian F. Platzer

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### ABSTRACT

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An elementary theory is developed for the supersonic flow around oscillating slender pointed bodies of revolution which accounts for body-shape and Mach-number effects. The frequency  $\omega$  is supposed to be small in comparison to U/L (U = free-stream velocity, L = body-length). The powers of  $\omega$  higher than one are neglected. By properly expanding Dorrance's solution with respect to the thickness ratio, an expression for the velocity-potential is obtained which is much handier for numerical evaluation. This solution is shown to be the low-frequency case of Adams-Sears' not-so-slender-body theory that, until now, could only be derived by Fourier or Laplace transform techniques.

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AERODYNAMICS ANALYSIS BRANCH AEROBALLISTICS DIVISION

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## DEFINITION OF SYMBOLS

Symbol	Definition
c	free-stream speed of sound
С	Euler's constant = 0.5772
F(x)	doublet-distribution
F(p)	Laplace-transform of F(x)
L	Laplace transform operator
M	free-stream Mach number
р	Laplace parameter
R(x)	body radius
t	time
U	free-stream velocity
x, r, θ	a system of $c$ ylindrical coordinates with x-axis in direction of free stream and with origin located at mean position of body nose (Fig. 1).
Z(x)	amplitude of downward displacement of body center line for harmonic motion
cot Q	$=\sqrt{M^2-1}$
λ	$= \cot^2 \alpha \cdot p^2 + 2 \frac{i\omega M}{c} p - \frac{\omega^2}{c^2}$
Λ	$= \cot^2 \alpha \cdot \frac{\partial^2}{\partial x^2} + 2 \frac{i\omega M}{c} \frac{\partial}{\partial x} - \frac{\omega^2}{c^2}$
κ	$= \frac{\omega}{c \cot^2 \alpha}$
ω	frequency of oscillation
μ	$= \frac{\omega U}{c^2 \cot^2 \alpha}$

## DEFINITION OF SYMBOLS (Cont'd)

Symbol Symbol	<u>Definition</u>
Φ(x, r, θ, t)	perturbation velocity potential for an arbitrary time-dependent motion
φ(x, r, θ)	amplitude of perturbation potential for harmonic motion
φ(p, r, θ)	Laplace transform of $\varphi(x, r, \theta)$ .

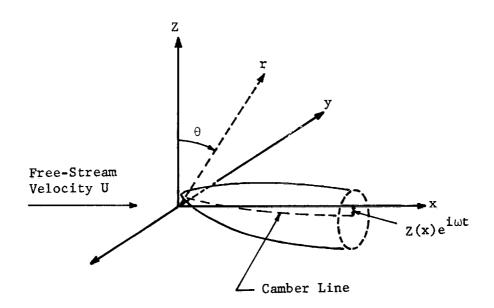


FIG. 1. CYLINDRICAL COORDINATES

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#### SUMMARY

An elementary theory is developed for the supersonic flow around oscillating slender pointed bodies of revolution which accounts for body-shape and Mach-number effects. The frequency  $\omega$  is supposed to be small in comparison to U/L (U = free-stream velocity, L = body-length). The powers of  $\omega$  higher than one are neglected. By properly expanding Dorrance's solution with respect to the thickness ratio, an expression for the velocity-potential is obtained which is much handier for numerical evaluation. This solution is shown to be the low-frequency case of Adams-Sears' not-so-slender-body theory that, until now, could only be derived by Fourier or Laplace transform techniques.

### INTRODUCTION

Current launch-vehicle developments have initiated increased interest in the prediction of aerodynamic forces on oscillating and/or deforming bodies of revolution because these forces are needed as input to both the dynamic stability problem and the aero-servo-elastic problem. Of special importance is the maximum dynamic pressure part of the ascent which is generally the lower supersonic part of the flight.

A theory readily available in the literature for determining the aerodynamic forces in this speed-range is the well-known Munk-Jones apparent-mass theory [1, 2] which was generalized to unsteady flow by Garrick [3] and Miles [4]. This theory - taking into account the crossflow only - gives no Mach-number dependence and is strictly applicable only to bodies of vanishingly small thickness. In recent years, several attempts have therefore been made for more rigorous theories applicable to not-so-slender bodies. We mention:

a. the theory of W. H. Dorrance [5] and extensions of his theory

to higher frequencies by D. L. Lansing [6] and R. B. Bond and B. B. Packard [7].

b. the "not-so-slender-body theory" of Adams-Sears as applied to unsteady supersonic flow by G. Zartarian and H. Ashley [8].

We should point out, however, that several other (essentially purely numerical) methods are possible [9, 10, 11, 12, 13, 14, 15, 16, 17] which may yield good agreement with experiment.

In this paper a quite elementary approach is derived which extends F. Keune's techniques for low aspect ratio bodies at zero angle of attack to unsteady flow and shows at the same time the essential equivalence of Dorrance's solution and the Adams-Sears theory.

### DORRANCE'S METHOD OF SOLUTION

Assuming harmonic time-dependence

$$\Phi(x, r, \theta, t) = \varphi(x, r, \theta) \cdot e^{i\omega t}$$

the linearized potential equation for pulsating flow reads

$$\cot^{2}\alpha \cdot \varphi_{xx} - \varphi_{rr} - \frac{1}{r} \varphi_{r} = -2 \frac{i\omega M}{c} \varphi_{x} + \frac{\omega^{2}}{c^{2}} \varphi. \tag{1}$$

A solution of this equation is the supersonic point-source

$$\phi_1(x, r) = -\frac{1}{2\pi R} e^{-i\mu(x-\xi)} \cdot \cos \kappa R$$
 (2)

where

$$R = \sqrt{(x - \xi)^2 - \cot^2\alpha \cdot r^2}$$

(e.g., I. E. Garrick, "Nonsteady Wing Characteristics," Vol. VII, High Speed Aerodynamics and Jet Propulsion, Princeton 1957, p. 677, equations 4-29).

The source-potential is obtained by distributing such sources along the x-axis

$$\phi_{s}(x, r) = -\frac{1}{2\pi} \int_{0}^{x-\cot\alpha \cdot r} \frac{F(\xi) \cos \left[\kappa \sqrt{(x-\xi)^{2} - \cot^{2}\alpha \cdot r^{2}}\right]}{\sqrt{(x-\xi)^{2} - \cot^{2}\alpha \cdot r^{2}}} e^{-i\mu(x-\xi)} d\xi.$$
(3)

From this solution the doublet-potential of the oscillating body of revolution can be derived by means of the operation

$$\varphi(\mathbf{x}, \mathbf{r}, \theta) = \cos \theta \cdot \frac{\partial \varphi_{\mathbf{s}}(\mathbf{x}, \mathbf{r})}{\partial \mathbf{r}},$$
 (4)

satisfying the equation

$$\cot^{2}\alpha \cdot \varphi_{xx} - \varphi_{rr} - \frac{1}{r} \varphi_{r} - \frac{1}{r^{2}} \varphi_{\theta\theta} = -2 \frac{i\omega M}{c} \varphi_{x} + \frac{\omega^{2}}{c^{2}} \varphi. \tag{5}$$

Laplace-transformation with respect to x

$$\bar{\varphi} (p, r) = \int_{0}^{\infty} e^{-px} \cdot \varphi(x, r) dx$$
 (6)

transforms equation (1) into

$$\bar{\varphi}_{rr} + \frac{1}{r} \bar{\varphi}_r - \lambda^2 \varphi = 0 \tag{7}$$

where

$$\lambda^2 = \cot^2 \alpha \cdot p^2 + 2 \frac{i\omega M}{c} p - \frac{\omega^2}{c^2}. \tag{7a}$$

A solution of equation (7) with the proper behavior at infinity is

$$\bar{\phi}_{S}(p, r) = -\frac{1}{2\pi} \bar{F}(p) \cdot K_{O}(\lambda r)$$
 (8)

$$\bar{F}(p) = L[F(x)]. \tag{9}$$

Expanding  $K_O(\lambda r)$  in terms of  $\lambda r$  and inserting into the Laplace-transformed equation (4) gives

$$\bar{\phi}(p, r, \theta) = -\frac{\cos \theta}{2\pi} \bar{F}(p) \left[ -\frac{1}{r} - \frac{\lambda^2 r}{2} (C + \ln \lambda) - \frac{\lambda^2 r}{2} \ln \frac{r}{2} + \right]$$

$$\frac{\lambda^2 r}{4} + \dots + 0(\lambda^4 r^3 \ln \lambda r)$$
 (10)

In a first approximation we retain only the first term of this expansion, thus obtaining after inversion

$$\varphi(x, r, \theta) = \frac{\cos \theta}{2\pi r} F(x). \tag{11}$$

This is the well-known slender-body solution. For a body whose axis performs harmonic oscillations  $Z(x)e^{i\omega t}$ , the boundary-condition reads

$$\lim_{r \to R(x)} \frac{\partial \varphi}{\partial r} = -\cos \theta \cdot \left[ U \frac{\partial Z(x)}{\partial x} + i\omega Z(x) \right] = -\cos \theta \cdot w(x). \tag{12}$$

The unknown doublet-distribution then is determined from

$$\lim_{r \to R(x)} \frac{\partial \varphi}{\partial r} = -\frac{F(x)}{2\pi R^2(x)} \cos \theta = -\cos \theta \cdot w(x). \tag{13}$$

Dorrance inserts this doublet-distribution back into equation (4) and restricts his solution to small frequencies, i.e., expands with respect to  $\omega$ , keeping only first-order terms. His solution therefore reads

$$\varphi(x, r, \theta) = \cos \theta \cdot \frac{\partial}{\partial r} \left[ -\frac{1}{2\pi} \int_{0}^{x-r} \frac{x-r \cot \alpha}{\int_{0}^{x-r} \frac{F(\xi) \left[1 - i\mu(x-\xi)\right]}{\sqrt{(x-)^2 - r^2 \cot^2 \alpha}}} d\xi \right], \quad (14)$$

which is evaluated by means of the substitution

$$\xi = x - \cot \alpha \cdot r \cdot \cosh u$$
.

 ${\tt J.}$  W. Miles criticized this procedure in Reference 19 as inconsistent and unnecessarily complicated.

If higher-order terms are sought, going beyond the slender-body solution, the Adams-Sears procedure [18] provides a consistent extension. It was applied to the oscillating body of revolution by Ashley and Zartarian [8]. We will therefore give a short recapitulation of the Adams-Sears theory.

### THE ADAMS-SEARS NOT-SO-SLENDER-BODY THEORY

In seeking a closer approximation, the higher-order terms of equation (10) have to be retained

$$\bar{\phi}(p, r, \theta) = -\frac{\cos \theta}{2\pi} \bar{F}(p) \left[ -\frac{1}{r} - \frac{\lambda^2 r}{2} (c + \ln \lambda) - \frac{\lambda^2 r}{2} \ln \frac{r}{2} + \frac{\lambda^2 r}{4} \right],$$
 (10)

and inversion gives after applying the Faltung theorem (e.g., Ref. 20, p. 124)

$$\phi(x, r, \theta) = \frac{\cos \theta}{2\pi r} F(x) - \frac{\cos \theta}{2\pi} \left[ -\frac{r\Lambda}{2} \left\{ F(x) \ln \cot \alpha + \frac{\partial}{\partial x} \int_{0}^{x} F(\xi) \ln (x - \xi) d\xi \right\} + \frac{r\Lambda}{4} \int_{0}^{x} \left\{ 2 - e^{-\frac{i\omega(x - \xi)}{c(M + 1)}} - e^{-\frac{i\omega(x - \xi)}{c(M - 1)}} \right\} \frac{F(\xi)}{x - \xi} d\xi +$$

$$- (\frac{r}{2} \ln \frac{r}{2}) \Lambda F(x) + \frac{r}{4} \Lambda F(x) \right],$$

$$(15)$$

where

$$\Lambda = \cot^2 \alpha \frac{\partial^2}{\partial x^2} + 2 \frac{i\omega M}{c} \frac{\partial}{\partial x} - \frac{\omega^2}{c^2} . \tag{16}$$

We are again interested in the small-frequency case and obtain from equation (15)

$$\phi(\mathbf{x}, \mathbf{r}, \theta) = \frac{\cos \theta}{2\pi r} F(\mathbf{x}) + \frac{\cos \theta}{4\pi} \mathbf{r} \cot^{2}\alpha \left[ F''(\mathbf{x}) \left( \ln \frac{\cot \alpha \cdot \mathbf{r}}{2\mathbf{x}} - \frac{1}{2} \right) + \right.$$

$$\left. + \int_{0}^{\mathbf{x}} \frac{F''(\mathbf{x}) - F''(\xi)}{\mathbf{x} - \xi} d\xi \right] +$$

$$\left. + \frac{i\omega U}{2\pi c^{2}} \mathbf{r} \cos \theta \cdot \left[ F'(\mathbf{x}) \ln \frac{\cot \alpha \cdot \mathbf{r}}{2\mathbf{x}} + \int_{0}^{\mathbf{x}} \frac{F'(\mathbf{x}) - F'(\xi)}{\mathbf{x} - \xi} d\xi \right].$$

$$\left. (17) \right.$$

# ELEMENTARY TRANSFORMATION OF DORRANCE'S SOLUTION FOR THE SLOWLY OSCILLATING POINTED BODY OF REVOLUTION

We will now demonstrate that the "higher-order terms" can be found by quite elementary steps. This was first shown by F. Keune for bodies of small aspect ratio at zero angle of attack [21]. We will extend his procedure to the case of the oscillating body of revolution.

For this purpose, we separate Dorrance's solution into a steadystate part

$$\cos \theta \cdot \frac{\partial}{\partial \mathbf{r}} \left[ -\frac{1}{2\pi} \int_{0}^{\mathbf{x}-\mathbf{r}} \frac{\cot \alpha}{\sqrt{(\mathbf{x}-\mathbf{\xi})^2 - \cot^2 \alpha \cdot \mathbf{r}^2}} \right]$$
 (18)

being the potential of a body of revolution at angle of attack and into an unsteady part

$$\cos \theta \cdot \frac{\partial}{\partial r} \left[ \frac{i\mu}{2\pi} \int_{0}^{x-r} \frac{\int_{0}^{x-r} \cot \alpha}{\int_{0}^{x-\xi} \left[ (x-\xi) \frac{d\xi}{2\pi} - \cot^{2}\alpha \cdot r^{2} \right]} \right]. \tag{19}$$

For the steady-state part we may immediately make use of Keune's developments for bodies at zero angle of attack. It is shown in Ref. 21 that in this case the velocity-potential may be written

$$\phi_{0}(x, r) = -\frac{1}{2\pi} \int_{0}^{x-r} \frac{\cot \alpha}{\sqrt{(x-\xi)^{2} - \cot^{2}\alpha \cdot r^{2}}} = \phi_{0}^{[0]} + \phi_{0}^{[II]} + \dots$$
(20)

where

$$\phi_0^{[0]} = \frac{F(x)}{2\pi} \ln r + \frac{F(x)}{2\pi} \ln \cot \alpha - \int_0^x F'(\xi) \ln [2(x-\xi)] d\xi,$$
 (20a)

the first term being the crossflow, the second and third term representing the spatial influence, and

$$\phi_{0}^{[II]} = \frac{\cot^{2}\alpha}{8\pi} r^{2} \cdot F''(x) \left[ \ln (r \cot \alpha) - 1 \right] + \frac{\cot^{2}\alpha}{8\pi} r^{2} \frac{\partial^{2}}{\partial x^{2}} \int_{0}^{x} F'(\xi) \ln \left[ 2(x-\xi) \right] d\xi.$$
(20b)

Inserting these expressions into equation (18) and performing the differentiation with respect to r gives for the steady-state part then

$$\cos \theta \cdot \frac{\partial}{\partial r} \left[ -\frac{1}{2\pi} \int_{0}^{\mathbf{x}-\mathbf{r}} \cot \alpha \frac{\mathbf{F}(\xi) \ d\xi}{\sqrt{(\mathbf{x}-\xi)^{2} - \cot^{2}\alpha \cdot \mathbf{r}^{2}}} \right] = \frac{\cos \theta}{2\pi \mathbf{r}} \mathbf{F}(\mathbf{x}) + \frac{\cos \theta}{4\pi} \mathbf{r} \cot^{2}\alpha \left[ \mathbf{F}''(\mathbf{x}) \left( \ln \frac{\cot \alpha \cdot \mathbf{r}}{2} - \frac{1}{2} \right) - \frac{\partial^{3}}{\partial \mathbf{x}^{3}} \int_{0}^{\mathbf{x}} \mathbf{F}(\xi) \ln (\mathbf{x}-\xi) \ d\xi \right]$$
(21a)

or in an alternative form

$$= \frac{\cos \theta}{2\pi r} F(x) + \frac{\cos \theta}{4\pi} r \cot^{2}\alpha \left[ F''(x) \left( \ln \frac{\cot\alpha \cdot r}{2x} - \frac{1}{2} \right) + \int_{0}^{x} \frac{F''(x) - F''(\xi)}{x - \xi} d\xi \right]. \tag{21b}$$

The unsteady part can likewise be transformed by means of the following relation which is easily shown to hold for pointed bodies

$$\frac{\partial}{\partial r} \int_{0}^{x-r} \frac{F(\xi) (x-\xi) d\xi}{\sqrt{(x-\xi)^{2} - \cot^{2}\alpha \cdot r^{2}}} = -\frac{\partial}{\partial x} \int_{0}^{x-r} \frac{F(\xi) \cot^{2}\alpha \cdot r d\xi}{\sqrt{(x-\xi)^{2} - \cot^{2}\alpha \cdot r^{2}}}.$$
(22)

The right-hand side of equation (22) can now be rewritten in the form

$$-\frac{\partial}{\partial x}\int_{0}^{x-r}\frac{\cot\alpha}{\sqrt{(x-\xi)^{2}-\cot^{2}\alpha\cdot r^{2}}}=-\cot^{2}\alpha\cdot r\frac{\partial^{2}}{\partial x^{2}}\int_{0}^{x-r}\frac{x-r\cot\alpha}{\int_{0}^{x-r}\cot\alpha}\frac{x-\xi}{\cot\alpha\cdot r}d\xi$$
(23)

and approximated asymptotically for small r in the integrand and in the upper limit (Ref. 22, p. 3).

$$-\cot^{2}\alpha \cdot r \frac{\partial^{2}}{\partial x^{2}} \int_{0}^{x-r} \frac{\cot \alpha}{F(\xi)} \cosh^{-1} \frac{x - \xi}{\cot \alpha \cdot r} d\xi = -\cot^{2}\alpha \cdot r \frac{\partial^{2}}{\partial x^{2}} \int_{0}^{x} F(\xi) \ln \frac{2(x - \xi)}{\cot \alpha \cdot r} d\xi.$$
(24)

The unsteady part therefore can be put into the form

$$\frac{i\mu}{2\pi}\cos\theta\frac{\partial}{\partial r}\int_{0}^{x-r}\frac{\cot\alpha}{\sqrt{(x-\xi)}\frac{d\xi}{-\cot^{2}\alpha \cdot r^{2}}} =$$

$$= +\frac{i\mu}{2\pi}\cot^{2}\alpha \cdot r\cos\theta\left[F'(x)\ln\frac{\cot\alpha \cdot r}{2} - \frac{\partial^{2}}{\partial x^{2}}\int_{0}^{x}F(\xi)\ln(x-\xi)d\xi\right]$$
(25a)

or again into the alternative form which is simpler for either analytical or numerical evaluation

$$\frac{i\mu}{2\pi}\cos\theta\cdot\frac{\partial}{\partial r}\int_{0}^{x-r}\frac{f(\xi)(x-\xi)d\xi}{\sqrt{(x-\xi)^{2}-\cot^{2}\alpha\cdot r^{2}}}=$$

$$=\pm\frac{i\mu}{2\pi}\cot^{2}\alpha\cdot r\cos\theta\cdot\left[F'(x)\ln\frac{\cot\alpha\cdot r}{2x}+\int_{0}^{x}\frac{F'(x)-F'(\xi)}{x-\xi}d\xi\right].$$
(25b)

Thus, Dorrance's solution is converted into the form

$$\phi(x, r, \theta) = \cos \theta \cdot \frac{\partial}{\partial r} \left[ -\frac{1}{2\pi} \int_{0}^{x-r} \frac{\cot \alpha}{\sqrt{(x-\xi)^2 - \cot^2 \alpha \cdot r^2}} \right]$$

$$= \frac{\cos \theta}{2\pi r} F(x) + \frac{\cos \theta}{4\pi} r \cot^2 \alpha \left[ F''(x) \left( \ln \frac{\cot \alpha \cdot r}{2x} - \frac{1}{2} \right) + \right]$$
 (26)

$$+\int_{0}^{x} \frac{F''(x) - F''(\xi)}{x - \xi} d\xi + \frac{i\omega U}{2\pi c^{2}} r \cos \theta \left[ F'(x) \ln \frac{\cot \alpha \cdot r}{2x} + \frac{i\omega U}{2\pi c^{2}} \right]$$

$$+\int\limits_{0}^{x}\frac{F'(x)-F'(\xi)}{x-\xi}d\xi\bigg].$$

Comparing equation (26) with equation (17) we find that we have obtained the Adams-Sears solution for small frequencies. Thus, by means of the elementary steps, equations (20) through (25), we have bridged the two approaches of Dorrance and Adams-Sears.

### CONCLUSIONS

The well-known slender-body concept [Munk, Ref. 1] which is also valid for oscillatory flow problems as shown by Garrick [3] and Miles [4] can be improved by seeking a closer approach to the full linearized solution.

Two such approaches have been published:

- a. the theory of W. H. Dorrance which starts from the basic doublet solution, equations (3) and (4),
- the not-so-slender-body theory of Adams-Sears, which implies Fourier - or Laplace - transformation techniques.

It should be mentioned that the Adams-Sears theory is the more general theory embracing the unified treatment of low-aspect-ratio wings, wing-body combinations, and bodies at subsonic, transonic, and supersonic speeds.

Restricting our considerations to the problem of slowly oscillating slender pointed bodies of revolution, we have obtained in this paper a quite elementary approach by applying and extending F. Keune's techniques for low aspect-ratio bodies at zero angle of attack. This solution, equation (26), was found, moreover, to be consistently extractable from Dorrance's solution and to be the low-frequency special case of the Adams-Sears solution. Having demonstrated in this way the essential equivalence of these two solutions, it will be obvious that the Adams-Sears theory is the more expedient one as it is more practical to evaluate for bodies of arbitrary meridian-profile.

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## A NOTE ON THE SOLUTION FOR THE SLOWLY OSCILLATING BODY OF REVOLUTION IN SUPERSONIC FLOW

### M. F. PLATZER

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